Key Engineering Materials Vols. 321-323 (2006) pp 1644-1648
online at http://www.scientific.net

Nonlinear Structural Dynamic Analyses of Micro Cantilever Beams Actuated by Electrostatic Force

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Keywords: Micro Cantilever Beam, Electrostatic Force, Modal Analysis, Stability Analysis, Design

Abstract. Nonlinear modeling method for the structural dynamic analysis of a micro cantilever beam actuated by electrostatic force is presented in this study. Static deflection is first obtained by solving nonlinear static equilibrium equation and the modal and the stability characteristics are calculated at the static equilibrium position. It is found that the amplitude and the frequency of the applied electrostatic voltage influence the stability of the structure significantly.

Introduction

Due to the recent development of manufacturing technology, a number of high technology products are made with MEMS devices. Typical examples of MEMS devices are micro HDD actuators, micro DLP mirrors and signal detecting devices for personal mobile terminals. Since electrostatic force is a surface force, it is very efficient to actuate small scale structures. Therefore, the electrostatic force is frequently employed to actuate many MEMS devices. Electrostatic force has characteristics different from general driving force because it changes nonlinearly by the response of structures. Due to the characteristics, responses vary with voltages, which generate electrostatic force. Thus, the method of analyzing microstructures undertaking electrostatic force needs to be developed.

A cantilever beam undertaking electrostatic force has been used in micro cantilever beams or combined structures. Examples of MEMS are actuators, switches, sensors and tweezers etc. A group of researchers investigated the response of micro cantilever beams undertaking electrostatic force. In Refs. [1-3], static response for a single cantilever beam that has a rectangle cross section was forecasted and the allowable voltage of the structure is studied. In Ref. [4], with the *van der Waals force* considered, the static response of a single cantilever beam is studied while the dynamic response was investigated in Ref. [5]. In these studies, however, variations of modal characteristics were not considered. In the present study, double micro cantilever beams undertaking electrostatic force (see Refs. [6, 7] for single micro cantilever beam) are analyzed. Unlike the existing literature results, static responses are forecasted with the nonlinear electrostatic force considered. Since the geometric shapes and the material properties are hard to be controlled during the manufacturing process, the uncertainty characteristics of MEMS structures are also considered.

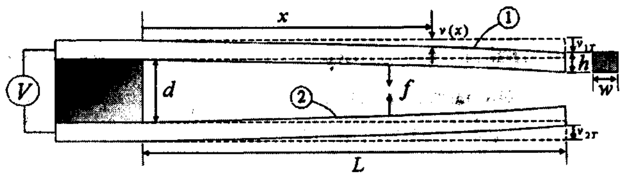


Fig. 1 Configuration of micro double cantilever beams actuated by electrostatic force.

Equations of motion

In this chapter, the equations of motion of double micro cantilever beam undertaking electrostatic force are derived. Fig. 1 shows a double micro cantilever beam undertaking electrostatic force. Based on Euler-Bernoulli beam theory, the differential equations of motion are obtained as follows:

$$\rho_1 \frac{\partial^2 v_1}{\partial t^2} + E_1 I_1 \frac{\partial^4 v_1}{\partial x_1^4} = f \tag{1}$$

$$\rho_2 \frac{\partial^2 v_2}{\partial t^2} + E_2 I_2 \frac{\partial^4 v_2}{\partial x_2^4} = -f \tag{2}$$

where f is the electrostatic force, which can be obtained (see Ref. [1]) as follows:

$$f = \frac{w \varepsilon_0 V^2}{2} \frac{1}{(d - v_1 + v_2)^2} \tag{3}$$

where w denotes the width of the beam, ε_0 denotes the permittivity of free space, V denotes the electrostatic voltage, d denotes the distance between the two beams, and v_1, v_2 denote the bending deflections of the two beams. Now, the bending deflection v can be approximated as follows:

$$v_\alpha(x,t) = \sum_{i=1}^{\mu_\alpha} \phi_i(x) q_i(t) \qquad (\alpha = 1,2) \tag{4}$$

With Eq. 4 being employed, the ordinary differential equations of motion can be obtained as follows:

$$\sum_{j=1}^{\mu_1} m_{ij}^1 \ddot{q}_{1j} + \sum_{j=1}^{\mu_1} k_{ij}^1 q_{1j} = r_i \quad (i = 1, \ldots, \mu_1) \tag{5}$$

$$\sum_{j=1}^{\mu_2} m_{ij}^2 \ddot{q}_{2j} + \sum_{j=1}^{\mu_2} k_{ij}^2 q_{2j} = -r_i \quad (i = 1, \ldots, \mu_2) \tag{6}$$

where

$$m_{ij}^\alpha \equiv \int_0^L \rho_\alpha \phi_i \phi_j dx \qquad k_{ij}^\alpha \equiv \int_0^L E_\alpha I_\alpha \phi_i'' \phi_j'' dx \qquad r_i \equiv \int_0^L f \phi_i dx \qquad (\alpha = 1,2)$$

From Eq. 5 and Eq. 6, the governing equations to obtain the static response can be derived as follows:

$$\sum_{j=1}^{\mu_1} k_{ij}^1 q_{1j} = r_i \quad (i = 1, \ldots, \mu_1) \tag{7}$$

$$\sum_{j=1}^{\mu_2} k_{ij}^2 q_{2j} = -r_i \quad (i = 1, \ldots, \mu_2) \tag{8}$$

The equilibrium position can be calculated from Eq. 7 and Eq. 8. The electrostatic force should be linearized to obtain a linear equation of motion, with which the modal characteristics can be obtained.

For the purpose, the electrostatic force is linearized (at the static equilibrium position v^*) as follows:

$$f \cong \frac{w\varepsilon_0 V^2}{2}\frac{d-3v^*}{(d-v^*)^3}+\frac{w\varepsilon_0 V^2}{(d-v^*)^3}(v_1-v_2) \tag{9}$$

$$v^* \equiv v_1^* - v_2^* \tag{10}$$

Numerical Analysis

In this chapter, based on the derived equations of motion, static, modal, and stability analysis of double micro cantilever beams undertaking electrostatic forces are performed. The numerical data are presented in table 1. Fig. 2 shows the dimensionless deflections of the free end versus applied voltage. It is shown that the dimensionless deflections increase as the voltage increase. The limit voltage of infinite static deflection can be also obtained. The limit voltage is reduced by decreasing the stiffness. Fig. 3 shows the variations of natural frequencies versus the applied voltages. In equal stiffness case, as the voltage increases, only the odd natural frequencies decrease. The differences between the first and the second natural frequencies are significantly large.

Table 1 Numerical data used for the simulation.

Notation	Description	Numerical data
ρ	Mass per unit length	9.320×10^{-9} kg/m
E	Young's modulus	55 GPa
I	Moment of inertia	1.3333 μm^4
L	Beam length	100 μm
w	Beam width	2.0 μm
ε_0	Free space permittivity	8.8542×10^{-12} F/m
V	Applied drive voltage	0 ~ 24 V
d	Gap between fixed tip	2.0 μm

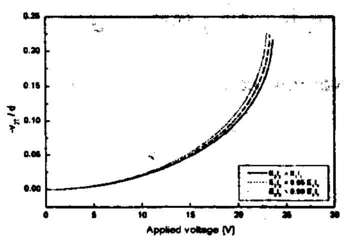
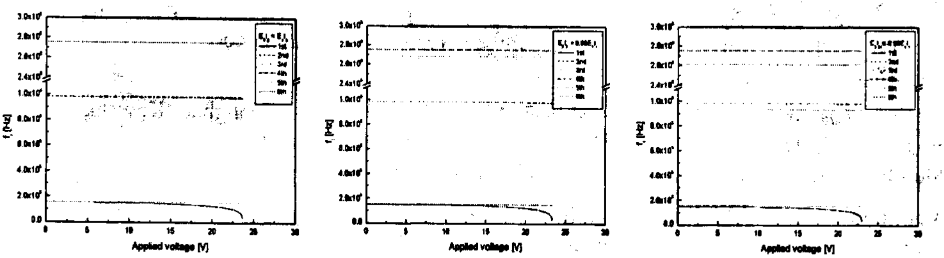
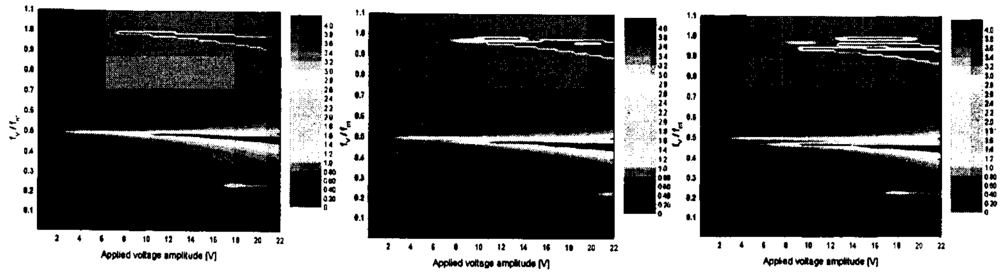


Fig. 2 Static deflections of beam tips versus the applied voltage.



(a) $E_2I_2 = E_1I_1$ (b) $E_2I_2 = 0.95E_1I_1$ (c) $E_2I_2 = 0.90E_1I_1$

Fig. 3 Variations of the lowest six natural frequencies versus the applied voltage.



(a) $E_2I_2 = E_1I_1$ (b) $E_2I_2 = 0.95E_1I_1$ (c) $E_2I_2 = 0.90E_1I_1$

Fig. 4 Variations of the dimensionless maximum dynamic response.

Fig. 4 shows dimensionless maximum responses versus the dimensionless applied frequency. The value $v_{\max}/d$ is exhibited in the figure. If the value is large, it can be considered as an unstable response. The maximum response becomes significant when f_V / f_{n1} reaches the value of 0.5 or 1.0. The instability intensity increases as the voltage increases. Also, the instability region moves toward lower frequency region as the voltage increases.

Conclusion

In this study, response and modal analyses are performed for double micro cantilever beams that are popularly employed in many MEMS structures. The static response is obtained by considering nonlinear electrostatic force and the limit voltage can be obtained from analysis results. Also, general modal and stability characteristics are obtained with modal and dynamic analysis results. Especially, the effects of bending stiffness asymmetry caused by MEMS manufacturing errors on the modal, the dynamic, and the instability characteristics are investigated in the present study. It is verified that the stiffness difference affects the dynamic response bigger than the static response. The results obtained in this study provide valuable information on designing micro actuators, switch and tweezers.

Acknowledgment

This research was supported by Center of Innovative Design Optimization Technology(iDOT), Korea Science and Engineering Foundation.

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